Contents

Chapter 4. Problem Solving in Geometry

Chapter 5. Problem Solving in Probability

To the Teacher

The person most closely associated with the teaching of problem solving in a mathematics context is George Polya. Polya suggests that problem solving involves finding a way around an obstacle to a desired end that is not immediately attainable. He points out that humans are by nature problem-solving animals and that the teaching of problem-solving skills should be one of the primary goals of education. He suggests that mathematics is an ideal medium through which to teach problem solving. The National Council of Teachers of Mathematics' position on problem solving is similar to this. Here are two quotes from page 23 of the well-known NCTM publication, *Curriculum and Evaluation Standards for School Mathematics*:

> *Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but* ***a process*** *that should permeate the entire program and provide the context in which concepts and skills can be learned.*
>
> *When problem solving becomes an integral part of classroom instruction and children experience success in solving problems, they gain confidence in doing mathematics and develop persevering and inquiring minds. They also grow in their ability to communicate mathematically and use higher-level thinking processes.*

One of the problem-solving strategies most often used in mathematics is looking for patterns. Chapter 1 of this book contains seven lessons in which solutions are derived through investigation of numerical patterns. Other chapters of this book contain problems that can best be solved by looking for patterns that are not numerical.

Chapter 2 contains lessons in which problem-solving situations are presented in a graphical context, and students solve these problems by careful investigation of the graphs. This chapter also includes two lessons in which students construct their own graphs and then use those graphs to solve problems.

Chapter 3 contains lessons in which information is presented in a picture or chart format, and students use this information to solve related problems. The lessons in this chapter involve such things as telephone bills, electric bills, time zones, and tournaments.

Chapter 4 involves problem solving in a geometric context. In our opinion, this area of mathematics is a rich source of problem-solving situations. Most textbooks do not approach geometry from this perspective. The typical textbook lesson usually

emphasizes the vocabulary aspect of this subject. This chapter contains lessons involving both two-dimensional geometry and three-dimensional geometry.

One of the key components of any learning activity is, of course, motivation. In the preparation of this book, every effort was made to select topics inherently interesting to youngsters and to build lessons around those topics. The lessons in Chapter 5 involve problem solving in a probability context, and probability is a topic that fascinates many young people.

—Using the Book—

The lessons in this book are meant to be used as a supplement to your textbook. It is often desirable to relate these lessons to the unit or topic being studied in class, but of course, some of the lessons will not have any direct relationship to those topics. This book does contain 40 lessons, so you could use one lesson each week during the school year.

We have made every effort to make the lessons easy to use. Most of the lessons are independent of each other, but when a particular lesson is a prerequisite for another lesson, that prerequisite is noted in the lesson plan. Most lessons involve some kind of worksheet, and some of the lessons require a transparency. Worksheet masters and transparency masters are provided. Worksheets and transparencies are keyed to individual lessons. For example, the transparency for Lesson 3-6 is labeled "Transparency 3-6" and the single worksheet for Lesson 3-10 is labeled "Worksheet 3-10." On the other hand, Lesson 3-5 has two worksheets and these worksheets are labeled "Worksheet 3-5-1" and "Worksheet 3-5-2." Each lesson contains a "Directions for the Teacher" section. In each case, this section contains specific suggestions on how you can teach the lesson, together with answers for individual problems.

In most cases, we suggest that you introduce the lessons to the class as a whole and then have students solve the problems on their own. You could also use these lessons with individual students as needed. Students could also complete the solutions in small groups rather than individually. More specifically, once a lesson is introduced in a large-group format, students could be assigned to groups of three or four and solutions could be determined in the groups.

LESSON 3-8

Taking Trips

Materials Needed

One copy of each of the following pages for each student: Worksheets 3-8-1 and 3-8-2

Directions for the Teacher

Initiate the lesson by asking students what is on the instrument panel of a car. Suggest that quite a lot can be learned by looking at the instrument panel. Distribute a copy of each worksheet page to each student and provide individual assistance as needed. Answers and comments about the questions follow.

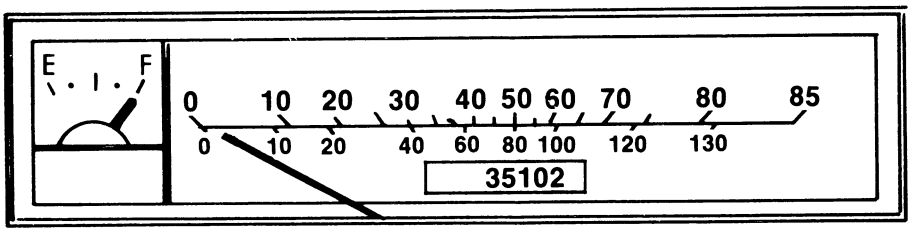
1. Most students immediately say yes, because the second picture shows that more than half a tank remains. However, the students should also consider the miles traveled. Looking only at the gas gauge, it is possible that the car was filled with gas several times on the trip, but since the trip was only 195 miles, they probably did not add gas to the tank.
2. 195 miles.
3. Assuming a constant speed of 65 mph (interstate speed limit), it would take about 3 hours.
4. Assuming that the car is driven about 10,000 or 12,000 miles a year, the car would be about three or four years old. However, many students respond in terms of how "old" the instrument panel looks.
5. No. They have already used more than half a tank of gas.
6. 203 miles.
7. Since there is only a little more than 1,000 miles on the odometer, it is probably fairly new. However, sometimes students will say it is old, because the odometer "rolled over."
8. It is impossible to tell, since the tanks may be of different sizes.
9. Since the car used about six gallons of gas, and since $203 \div 6 \approx 34$, the gas mileage for the trip was about 34 miles per gallon.

Name _____ Date _____

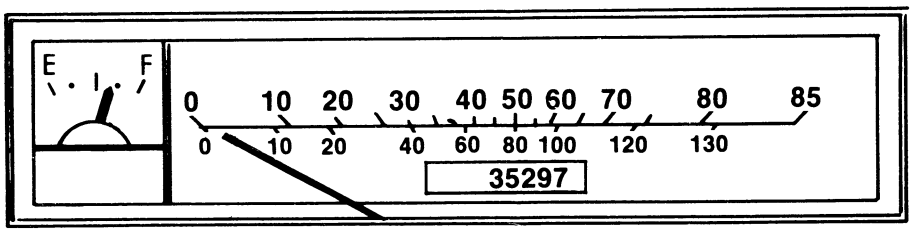
Worksheet 3–8–1

Taking Trips

The Jones and Smith families are friends and visit each other regularly. One day, as the Jones family left home on their way to the Smith house, the instrument panel on their car looked like this:



When they arrived at the Smith house, their instrument panel looked like this:



1. Do you think they could return home without adding gas to their tank? Why?

2. How far is it between the two houses? _____

3. Most of this trip was on an interstate highway. About how long do you think it took? _____

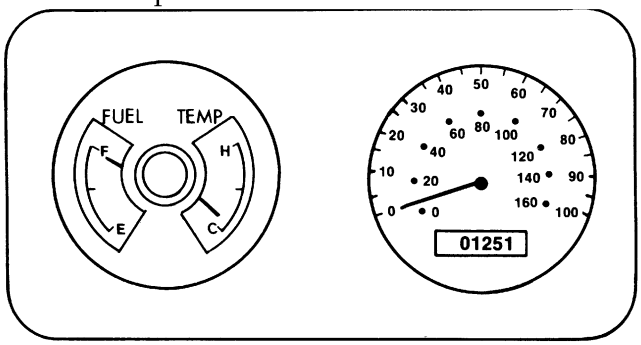
4. About how old do you think the Jones car is? _____

Name _____ Date _____

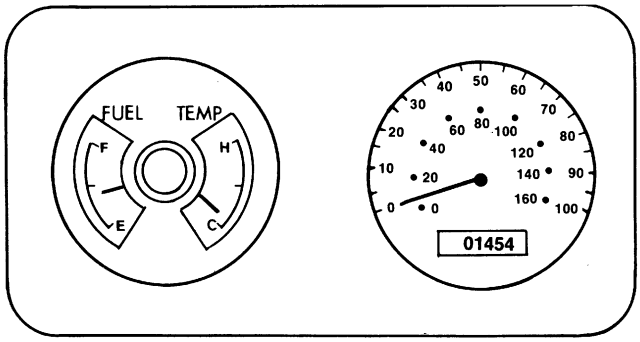
Worksheet 3–8–2

Taking Trips

About a month later, the Smith family went to visit the Jones family. When they left home, the instrument panel on their car looked like this:



When they arrived at the Jones home, it looked like this:



5. Do you think the Smith family could return home without adding gas to their tank? Why?

   _____

6. How far is it between the two houses (according to the odometer on the Smith car)? _____

7. About how old do you think the Smith car is? Why? _____

   _____

8. Which car used the least gas for the trip? _____

9. The gas tank on the Smith car holds 10 gallons. What was their approximate gas mileage for the trip? _____

LESSON 3-9

Time Zones

Materials Needed

One copy of the following pages for each student: Worksheets 3-9-1 and 3-9-2

Directions for the Teacher

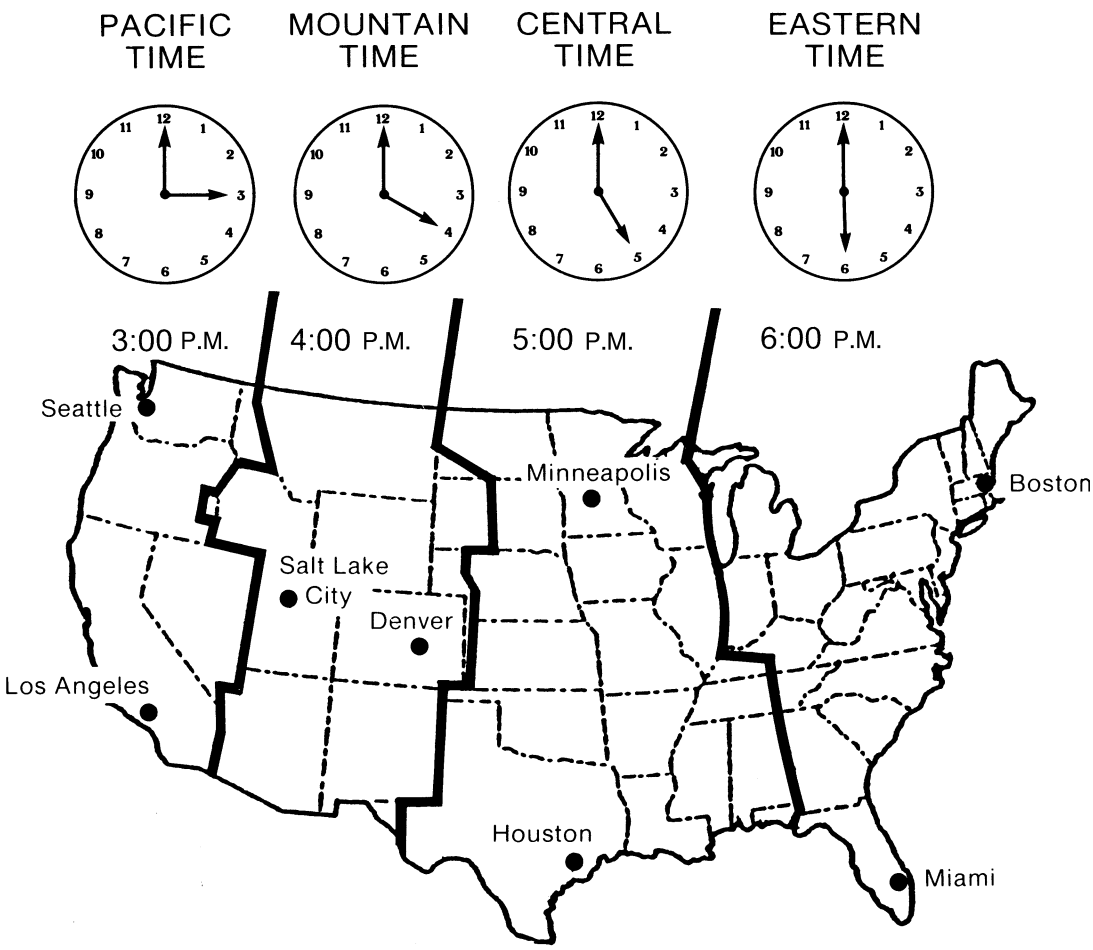
Distribute a copy of each worksheet page to each student. Direct the students' attention to the picture on the top of the first worksheet page. Mention that this illustrates that when it is 3:00 P.M. Pacific Time, is is 4:00 P.M. Mountain Time, 5:00 P.M. Central Time, and 6:00 P.M. Eastern Time. Ask what time it would be in each of the other zones if it were 11:00 A.M. Eastern Time (8:00 A.M. PT, 9:00 A.M. MT, and 10:00 A.M. CT). Ask what time it would be in each of the other zones if it were 10:23 Pacific Time (11:23 P.M. MT, 12:23 A.M. CT, 1:23 A.M. ET). Have the students proceed to the problems on the worksheets. Provide individual help as needed. Correct answers follow:

1. 9 A.M.
2. 2 P.M.
3. 6 A.M.
4. 11 P.M.
5. 1 A.M.
6. 1 P.M.
7. 4 hrs. 8 mins.
8. 4 hrs. 49 mins.

Name _____ Date _____

Worksheet 3-9-1

Time Zones



1. If it is 7:00 A.M. in Seattle, what time is it in Minneapolis? _____
2. If it is 4:00 P.M. in Boston, what time is it in Denver? _____
3. If it is 9:00 A.M. in Miami, what time is it in Los Angeles? _____
4. If it is 1:00 A.M. in Houston, what time is it in Seattle? _____
5. If it is 11:00 P.M. in Salt Lake City, what time is it in Boston? _____

Name _____ Date _____

Worksheet 3–9–2

Time Zones

6. If it is 10:00 A.M. in Los Angeles, what time is it in Miami? _____

When you fly across time zones, departure and arrival times are always given for the appropriate time zone.

7. If you leave Boston at 6:01 P.M. and take a direct flight to Minneapolis that arrives at 9:09 P.M., how long did your flight last? _____

8. Your plane leaves Seattle at 6:30 A.M. and arrives in Houston at 1:19 P.M. How long did the flight last? _____

LESSON 3-10

Baseball Games

Prerequisite Lessons

Lessons 3-1 and 3-2

Materials Needed

One copy of Worksheet 3-10 for each student

Directions for the Teacher

The problems on the worksheet are quite difficult, so initially a couple of different approaches to each problem will be described.

Problem 1, Approach 1

It is desirable to first try to determine who plays the Cubs. Since Harry picked the Cubs, Braves, and Giants, the Cubs could not have played those two teams. A comparable investigation could be made of Mary's selections and Cary's selections. Here is a table that illustrates these situations.

Team	Possible Opponents				
Cubs	Braves	Cardinals	Dodgers	Giants	Reds
	no		no	no	no

These "no's" result from the elimination described above. Thus, the Cubs must play the Cardinals. It was important to investigate the Cubs first, because they were picked by the most people (all three people). Next we must choose a team that was chosen by two people (Braves and Dodgers). Let's arbitrarily choose the Braves. Since we know from above that the Cubs play the Cardinals, the Braves couldn't play either

of those teams. Using that information and eliminating other teams that were jointly selected with the Braves results in the following table:

Team	Possible Opponents				
Braves	Cardinals	Cubs	Dodgers	Giants	Reds
	no	no	no	no	

Thus, the Braves played the Reds. Since the other four teams have been eliminated, the Dodgers must have played the Giants. Here is a summary of these results.

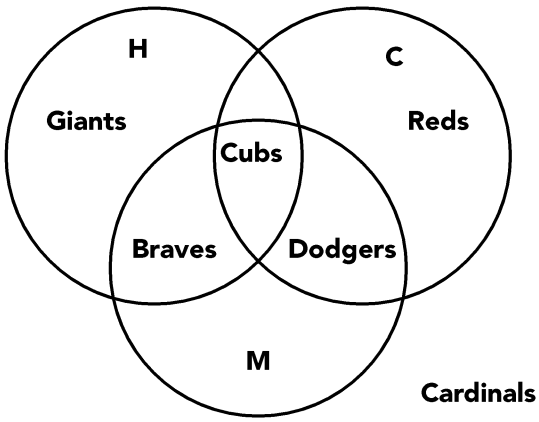
Cubs played Cardinals

Braves played Reds

Dodgers played Giants

Problem 1, Approach 2

A Venn diagram is helpful. Let H represent Harry's picks, let M represent Mary's picks, and let C represent Cary's picks.



This diagram results from the way the picks were made. More specifically, the Cubs were picked by all three, the Braves were picked by Harry and Mary, the Dodgers were picked by Mary and Cary, the Giants were picked by only Harry, the Reds were picked by only Cary, and the Cardinals were not picked at all. The diagram shows that the Cubs did not play the Braves, the Giants, the Dodgers, and the Reds. Thus, the Cubs must have played the Cardinals. Next, examine the possibilities for the Braves. Since the Cubs played the Cardinals, those two teams are eliminated. Since Harry picked both the Braves and Giants, the Braves could not have played the Giants. Since Mary picked both the Braves and the Dodgers, the Braves could not have played

the Dodgers. Thus, the Braves played the Reds. That leaves the Dodgers and Giants, and they must have played each other.

Problem 2, Approach 1

(This will be very much like approach 1 for problem 1.) Since the Pirates were picked by all three people, it would be desirable to start with them. For instance, the Braves, Cubs, and Giants can be eliminated, since Mary picked those teams and the Pirates. Other possible opponents can be eliminated in a similar way. Here is the resulting table.

Team	Possible Opponents						
Pirates	Braves no	Cardinals no	Cubs no	Dodgers no	Giants no	Mets	Reds no

Thus, the Pirates play the Mets. Next, pick a team picked by two people (Cubs, Giants, Reds). Arbitrarily, let's pick the Giants. Build another table.

Team	Possible Opponents						
Giants	Braves no	Cardinals no	Cubs no	Dodgers	Mets no	Pirates no	Reds no

The Pirates and Mets can be eliminated, since they play each other. Others could be eliminated using the manner described for the previous table. Thus, the Giants played the Dodgers. Since the Reds were also picked twice, we could build a comparable table. If we did that, we would conclude that the Reds played the Braves. Since only the Cubs and the Cardinals are left, they must have played each other. Here is a summary of these results:

Pirates played Mets

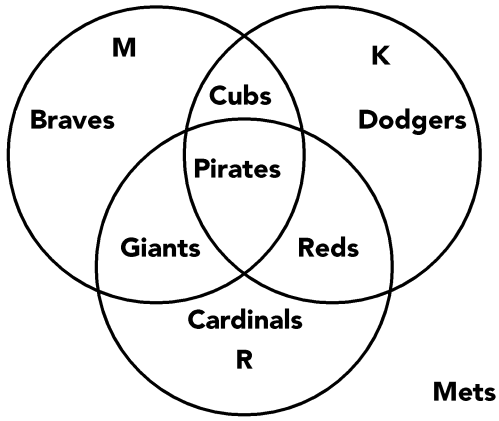
Giants played Dodgers

Reds played Braves

Cubs played Cardinals

Problem 2, Approach 2

(This will be very much like approach 2 for problem 1.) A Venn diagram is helpful. Let M represent May's picks, let K represent Kay's picks, and let R represent Ray's picks.



Using a process of elimination like the one described in approach 2 for problem 1, the Pirates played the Mets. Since the Giants were picked by both May and Ray, the Giants didn't play the Braves, Cubs, Pirates, Reds, and Cardinals. The Giants didn't play the Mets, since the Pirates did. Thus, the Giants played the Dodgers. In a similar manner, it can be determined that the Reds must have played the Braves and the Cubs must have played the Cardinals.

As mentioned above, the two problems on the worksheet are quite difficult. We recommend that you distribute the worksheet, that students start working on these problems in groups of three or four, and that these students be encouraged to solve the problems within their groups. If the students become very frustrated, you may want to give some hints that might lead to approaches like the ones given earlier.

Name _____ Date _____

Worksheet 3-10

Baseball Games

1. On one day, three baseball games were played. The teams that played in those games were the Braves, Cardinals, Cubs, Dodgers, Giants, and Reds. Harry, Mary, and Cary picked the winners of those games. The choices for each person are as follows:

Harry: Braves, Cubs, Giants

Mary: Braves, Cubs, Dodgers

Cary: Cubs, Dodgers, Reds

Find out which teams played which other teams.

Hint: Name some teams that you know *didn't* play the Cubs.

2. On another day, four baseball games were played. The teams that played in those games were the Braves, Cardinals, Cubs, Dodgers, Giants, Mets, Pirates, and Reds. May, Kay, and Ray picked the winners of those games. The choices for each person are as follows:

May: Braves, Cubs, Giants, Pirates

Kay: Cubs, Dodgers, Pirates, Reds

Ray: Cardinals, Giants, Pirates, Reds

Find out which teams played which other teams.